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# PARAMETRIC OSCILLATIONS OF A MEMBRANE IN AN ALTERNATING ELECTRIC FIELD†

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**Axisymmetric oscillations of a circular conducting elastic membrane placed between the grounded plates of a capacitor and subjected to the action of an alternating voltage are investigated. Approximate expressions are obtained in the quasistationary approximation of the equations of electrodynamics for the potential of the electric field between the plates and a self-consistent integro-partial differential equation in the oscillations of the membrane. A solution of the corresponding boundary-value problem is constructed and a qualitative analysis of the parametrically excited oscillations is carried out. The conditions for stability and instability of the natural oscillations of different modes are obtained in terms of the electromechanical parameters**

## 1. INITIAL ASSUMPTIONS AND FORMULATION OF THE PROBLEM

Consider the oscillations of the electromechanical system shown schematically in Fig. 1 (the front or side view). The system consists of a plane capacitor, the distance between the plates 1 (the lower plate) and 2 (the upper plate) of which is $2h$. We will assume that the outer plates are grounded, i.e. their potential is zero. An elastic membrane is placed symmetrically between the plates at the same distance $h$ from the upper and lower plates. An electric voltage is applied to the membrane, the potential of which with respect to the plates is $U=U_0\cos\Omega t$, where $U_0$ and $\Omega$ are constants characterizing the amplitude and frequency, respectively.

We will make the following simplifying assumptions regarding the geometrical and electromechanical properties of the system:

1. the membrane and the capacitor plates are circles of radius $a$ (seen from above), where, by assumption, $h \ll a$, which enables us to neglect edge effects [1];
2. they are ideal conductors, and the permittivity of the medium in the cylindrical regions $D_1$ and $D_2$ is taken to be unity (to fix our ideas);
3. the inequality $a\Omega \ll c$ is satisfied, where $c$ is the velocity of light, which enables us to neglect displacement currents and to confine ourselves to the quasistatic approximation to describe the electric field [2, 3];
4. the membrane is assumed to be clamped (restrained) along the contour and is ideal;
5. to simplify the calculations the displacements of the membrane are assumed to be axisymmetrical, i.e. they are independent of the polar angle.

The formulation of the problem is as follows. At the initial instant of time $t=0$ points of the membrane are given axisymmetrical displacement and velocity distributions. It is required to obtain the motion of the membrane when $t>0$ taking the applied voltage $U(t)$ into account.

In order to determine the motion of the current-conducting membrane we must obtain the distribution of the electric-field forces acting on it. These forces can be calculated [2, 3] if we

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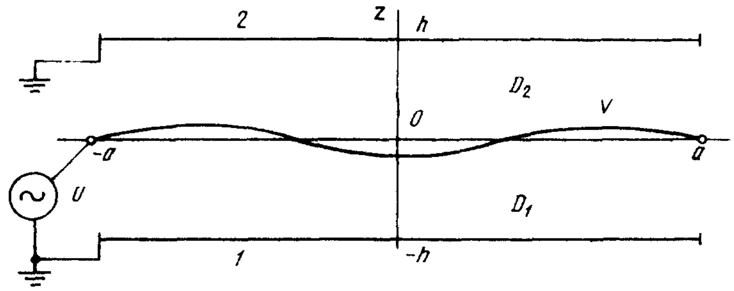

Fig. 1.

know the electric-field potential $u_1$ and $u_2$ in the regions $D_1$ and $D_2$, respectively. We will now determine these in the required quasistatic approximation for the electric field.

## 2. SOLUTION OF THE PROBLEM OF ELECTROSTATICS AND THE DETERMINATION OF THE FORCES ACTING ON THE MEMBRANE

In view of assumption 3 the electric field can be described by the equations of electrostatics. By assumption 1 the electric field is concentrated in the cylindrical region $D = D_1 \cup D_2$ between the capacitor plates, and hence we can neglect edge effects (see the "Rogovskii capacitor" [1]).

We will introduce a cylindrical system of coordinates whose centre coincides with the centre of the undeformed membrane. The $z$ axis is directed perpendicular to its plane (see Fig. 1). As was mentioned above, we will denote the unknown required electric-field potentials in the regions $D_1$ and $D_2$ by $u_1$ and $u_2$, where $u_{1,2}(r, z, t)$. We will denote the required symmetrical displacement of points of the membrane by $V = V(r, t)$.

The potentials $u_{1,2}$ are defined as the solutions of the following Dirichlet problems [2, 3]

$$\Delta u_1 = 0, \quad (r,z) \in D_1 = \{r,z\colon\ 0 \leqslant r < a,\ V > z > -h\} \tag{2.1}$$
$$u_1(r,-h,t) = 0, \quad u_1(r,V,t) = U_0 \cos\Omega t$$

$$\Delta u_2 = 0, \quad (r,z) \in D_2 = \{r,z\colon\ 0 \leqslant r < a,\ h > z > V\} \tag{2.2}$$
$$u_2(r,h,t) = 0, \quad u_2(r,V,t) = U_0 \cos\Omega t$$

Here $\Delta = r^{-1}\partial(r\partial/\partial r)/\partial r + \partial^2/\partial z^2$ is the two-dimensional Laplace operator in cylindrical coordinates (there is no dependence on the polar angle), the value of $V = V(r, t)$ in the boundary conditions is not known, and the time $t$ occurs as a parameter. The boundary conditions for $r = a$ are unimportant in view of assumption 1. Moreover, the solution $u(r, z, t)$ must be bounded as $r \to 0$: $|u| \leqslant M < \infty$.

Assuming the displacements of the membrane $V$ to be fairly small, i.e. $V/h \ll 1$ (and all the more $V/a \ll 1$), we seek solutions of problems (2.1) and (2.2) in the form of expansions in powers of $V$. The use of standard procedures of the method of perturbations becomes more convenient if we make the replacement $V \to \varepsilon V$, where $\varepsilon$ is a numerical parameter, and after carrying out formal expansions in powers of $\varepsilon$ we put $\varepsilon = 1$. For further analysis of the main qualitative properties of the motion of the system it turns out to be sufficient to confine ourselves to the first approximation $u = u_{1,2} = u_{1,2}^{(0)} + \varepsilon u_{1,2}^{(1)} + \varepsilon^2 \ \ldots \ (\varepsilon = 1)$. The unknown coefficients $u^{(0)}$, $u^{(1)}$, ... are obtained by successive solution of the corresponding boundary-value problems. To determine the unknown $u_{1,2}^{(0)}$ we have, by (2.1) and (2.2) in the regions $D_{1,2}^{(0)}$, the problems

$$\Delta u_{1,2}^{(0)} = 0, \quad u_{1,2}^{(0)}(r, \mp h, t) = 0$$
$$D_{1,2}^{(0)} = D_{1,2}|_{V=0}, \quad u_{1,2}^{(0)}(r,0,t) = U_0 \cos\Omega t$$

which do not contain the unknown $V$ in the boundary conditions. Taking into account the condition for the solution $u^{(0)}$ to be bounded as $r \to 0$, we obtain the expressions

$$u_{1,2}^{(0)} = U_0(1 \pm zh^{-1})\cos\Omega t \tag{2.3}$$

which are independent of the $r$ coordinate. These correspond to the case of an undeformed membrane.

To determine the unknown functions $u_{1,2}^{(1)}$ in the corresponding undeformed regions $D_{1,2}^{(0)}$ we require solutions of the boundary-value problems containing the unknown function $V = V(r, t)$

$$\Delta u_{1,2}^{(0)} = 0, \quad u_{1,2}^{(1)}(r, \mp h, t) = 0, \quad u_{1,2}^{(1)}(r, 0, t) = \mp Vh^{-1}U_0 \cos\Omega t$$

Here the functions $u_{1,2}^{(1)}$ must be bounded as $r \to 0$. Since when $h/a \ll 1$ we can neglect edge effects at the boundary $r = a$ [1], we can formulate arbitrary boundary conditions for $u_{1,2}^{(1)}$. Since the membrane is rigidly clamped along the edge ($V(a, t) = 0$), in the case considered it is also convenient to put $u^{(1)}(a, z, t) = 0$.

We will assume for the present that the function $V(r, t)$ is known and is fairly smooth. Then the required functions $u_{1,2}^{(1)}(r, z, t)$ are defined uniquely in the form of series

$$u_{1,2}^{(1)}(r,z,t) = \mp \frac{U_0}{h}\cos\Omega t \sum_{n=1}^{\infty} c_n \frac{\operatorname{sh}\gamma_n(h \pm z)a^{-1}}{\operatorname{sh}(\gamma_n h a^{-1})} R_n(r) \tag{2.4}$$

$$R_n(r) = J_0(\gamma_n r a^{-1}), \quad n = 1, 2, \ldots$$

$$\gamma_n \simeq (n - 1/4)\pi + 0{,}05\pi(4n-1)^{-1} + O(n^{-2}), \quad \gamma_{n+1} \simeq \gamma_n + \pi + O(n^{-2})$$

Here $J_0$ is a Bessel function of zero order, and the eigenvalues of the problem $\gamma_n$ are the roots of the equation $J_0(\gamma) = 0$ ($\gamma_1 = 2.4048$, $\gamma_2 = 5.5201$, $\gamma_3 = 8.6537$, $\gamma_4 = 11.7915$, $\gamma_5 = 14.9309$, etc. [4]). The coefficients $c_n$ of the series in (2.4) are Fourier coefficients of the function $V(r, t)$ in an orthogonal system $\{R_n(r)\}$ with weight $r$

$$c_n = c_n(t) = \frac{1}{\| R_n \|^2} \int_0^a V(\rho, t) R_n(\rho)\rho d\rho \tag{2.5}$$

$$\| R_n \|^2 = \int_0^a J_0^2\left(\gamma_n \frac{\rho}{a}\right)\rho d\rho = \frac{a^2}{2} J_1^2(\gamma_n)$$

As a result, we obtain approximate expressions, apart from quadratic terms in $\| V \|$ for the required potentials $u_{1,2}$ Taking (2.3) and (2.4) into account we conclude that they have the following form in regions $D_1$ and $D_2$, respectively

$$u_{1,2}(r,z,t) = U_0\left(1 \pm \frac{z}{h}\right)\cos\Omega t \mp \frac{U_0}{h}\cos\Omega t \sum_{n=1}^{\infty} c_n(t) \frac{\operatorname{sh}\gamma_n(h \pm z)a^{-1}}{\operatorname{sh}(\gamma_n h a^{-1})} J_0\left(\gamma_n \frac{r}{a}\right)$$

$$(r, z) \in D_{1,2} \tag{2.6}$$

where $c_n(t)$ are linear functionals of $V$ (2.5).

To determine the surface forces acting on the conducting membrane we must calculate the electric field strength $\mathbf{E}_{1,2}$ in the regions $D_{1,2}$ respectively from the formulae $\mathbf{E}_{1,2} = -\operatorname{grad} u_{1,2}$. The distribution of the forces, i.e. the normal pressure, is defined as follows [2, 3]:

$$P = \left.\frac{E_2^2 - E_1^2}{8\pi}\right|_{z=V} = \frac{U_0^2 \cos^2\Omega t}{4\pi a h^2} \sum_{n=1}^{\infty} c_n(t)\gamma_n \operatorname{cth}\left(\gamma_n \frac{h}{a}\right) J_0\left(\gamma_n \frac{r}{a}\right) \tag{2.7}$$

In deriving (2.7) we only took into account terms that are linear in $V$ and the fact that the representations (2.6) in this approximation hold in the initial regions $D_{1,2}$. It follows from (2.7) that when $V \equiv 0$ the normal pressure $P \equiv 0$, since $c_n(t) \equiv 0$, $n = 1, 2, \ldots$. Note that this is due to the symmetrical position of the membrane (at equal distances $h$ from the plates).

## 3. THE CONSTRUCTION AND SOLUTION OF THE BOUNDARY-VALUE PROBLEM DESCRIBING THE AXISYMMETRICAL MOTIONS OF THE MEMBRANE

Using expression (2.7) for the distributed forces $P$ which are orthogonal to the surface we can write the following equation of motion of the membrane [5]

$$\mu \ddot{V} - T\Delta V = P, \quad V = V(r,t), \quad r \leqslant a \tag{3.1}$$

$$P = P(r,t,[V]) \equiv \frac{U_0^2 \cos^2 \Omega t}{4\pi a h^2} \int_0^a G(r,\rho) V(\rho,t) d\rho$$

$$G(r,\rho) = \sum_{n=1}^{\infty} \gamma_n \operatorname{cth}\left(\gamma_n \frac{h}{a}\right) \| R_n \|^{-2} R_n(r) R_n(\rho) \rho$$

The dots denote differentiation with respect to time $t$, $\Delta = r^{-1}\partial(r\partial/\partial r)/\partial r$ is the one-dimensional Laplace operator with respect to $r$, $\mu$ is the surface density, and $T$ is the tension of the membrane. There is no dependence on the polar angle in view of assumption 5 in Section 1. Note that the state of the membrane is described by the integro-partial differential equation (3.1). The right-hand side of the equation contains a Fredholm-type operator, integral with respect to $r$.

It is required to obtain a solution of Eq. (3.1) satisfying the clamping and boundedness conditions

$$V(a,\, t) = 0, \quad |V(r,\, t)| \leqslant M < \infty, \quad r \to 0 \tag{3.2}$$

and also the specified axisymmetrical initial conditions

$$V(r,\, 0) = f(r), \quad \dot{V}(r,\, 0) = g(r) \tag{3.3}$$

The functions $f$ and $g$ are assumed to be consistent with conditions (3.2) and fairly smooth, so that a strong solution [5] of problem (3.1)–(3.3) exists.

It is natural to seek this solution in the form of expansions in the system of functions $R_n(r) = J_0(\gamma_n r a^{-1})$, orthogonal with weight $r$

$$V = V(r,t) = \sum_{n=1}^{\infty} V_n(t) R_n(r) \tag{3.4}$$

Using standard methods of mathematical physics [5] we obtain the following second-order ordinary differential equations with periodic Mathieu-type coefficients [6] for the coefficients $V_n(t)$ (3.4)

$$\ddot{V}_n + (\omega_n^2 - \nu_n^2 \cos^2 \Omega t) V_n = 0, \quad n = 1, 2, \ldots \tag{3.5}$$

$$\omega_n^2 = \gamma_n^2 T (\mu a^2)^{-1}, \quad \nu_n^2 = U_0^2 (4\pi a h^2 \mu)^{-1} \gamma_n \operatorname{cth}(\gamma_n h a^{-1})$$

Here $\omega_n$ are the natural frequencies of a circular membrane undergoing axisymmetrical oscillations. The parameters $\nu_n$ also have the dimensions of frequency, where $\nu_n \sim \sqrt{(\gamma_n)} \sim$

$\sqrt{(n)}$. Hence $\nu_n^2\omega_n^{-2} \sim n^{-1}$ as $n \to \infty$, i.e. the terms $\omega_n^2 V_n$ are the "principal" terms in Eq. (3.5) for sufficiently large $n$. The functions $V_n$ must satisfy the initial conditions, which follow from (3.3)

$$V_n(0) = f_n, \quad f_n = (f(r), R_n(r))_r \| R_n \|^{-2} \tag{3.6}$$
$$V_n^{\cdot}(0) = g_n, \quad g_n = (g(r), R_n(r))_r \| R_n \|^{-2}$$

It is assumed that the coefficients $f_n$ and $g_n$ decrease fairly rapidly as $n \to \infty$.

Hence, the boundary-value problem (3.1)–(3.3) reduces to the solution of a denumerable system of Cauchy problems (3.5) and (3.6) for linear equations with periodic coefficients, the frequency of variation of which is $2\Omega$. Note that the effect of the external electric field is determined by the square of the voltage $U^2 = U_0^2 \cos^2 \Omega t$. If $U_0 = 0$ we have $\nu_n = 0$ and the equations can be integrated in explicit form. The solution obtained describes the free axisymmetrical oscillations of a circular membrane. When $U_0 \neq 0$, $\Omega \neq 0$ the required solution $V_n(t)$, $n = 1, 2, \ldots$ can be written using Mathieu functions [6]. The case when $\Omega = 0$ a constant voltage) is also of interest (see below).

## 4. A QUALITATIVE ANALYSIS OF THE MOTION OF THE MEMBRANE

### 4.1. *The stability of the position of equilibrium and of the motions of the membrane in a constant electric field*

In Eqs (3.5) we put $\Omega = 0$; we obtain the denumerable system

$$V_n^{\cdot\cdot} + (\omega_n^2 - \nu_n^2) V_n = 0, \quad n = 1, 2, \ldots \tag{4.1}$$

The stability, in the linear approximation, of the oscillations of the membrane and of its position of equilibrium $V_n \equiv 0$, $V_n^{\cdot} \equiv 0$ follows from (4.1) if $\omega_n > \nu_n$ for all $n \geqslant 1$. In the initial physical variables the condition $\omega_n > \nu_n$ can be written as follows:

$$(U_0 / h)^2 < 4\pi T a^{-1} \gamma_n \operatorname{th}(\gamma_n h a^{-1}), \quad n = 1, 2, \ldots \tag{4.2}$$

Thus, stability in the linear approximation occurs if the electric field strength $U_0/h$ averaged over $z$) is sufficiently small compared with the parameter $(T/a)^{1/2}$ characterizing the tension of the membrane. Owing to the monotonicity of the right-hand side of inequality (4.2) with respect of $\gamma_n$ it is sufficient for this to be satisfied when $n = 1$, i.e. for $\gamma_1$. Since we have the strong inequality $ha^{-1} \ll 1$, condition (4.2) takes the following form when $n = 1$

$$(U_0 / h)^2 < 4\pi\gamma_1^2 (h/a) T a^{-1} \quad (\gamma_1^2 \simeq 5.7831) \tag{4.3}$$

The oscillations of the membrane lose their stability and become exponentially unstable if inequalities inverse to (4.2) are satisfied for certain values of $n = 1, 2, \ldots, n^*$ In particular, the loss of stability with respect to the fundamental first mode occurs when the inverse inequality to (4.3) is satisfied

$$U_0^2 a^2 (4\pi h^3 T) > \gamma_1^2 \simeq 5.7831 \tag{4.4}$$

Inequality (4.4) is similar in its form and physical meaning to the condition for a charged liquid drop which is in equilibrium under the action of surface-tension forces to be unstable (Rayleigh, see [7]).

The presence of linear dissipation means that modes of oscillation that are stable in the linear approximation become asymptotically (exponentially) stable. Exponentially stable modes remain such. Critical cases, when instead of strict inequalities we have equalities, require additional investigations taking the non-linearity into account. Note that the presence of a constant electric field leads to a reduction in the frequency of oscillations of the membrane by (4.1).

4.2. *Instability of the parametric oscillations of a membrane in an alternating electric field*

We will reduce Eqs (3.5) to the standard form of Mathieu equations [6]. To do this we will introduce dimensionless time $\tau$ and dimensionless parameters $\alpha_n$, $\beta_n$ by the following formulae

$$\tau = \Omega t, \quad \alpha_n = \omega_n^2\Omega^{-2}(1 - \tfrac{1}{2}\nu_n^2\omega_n^{-2}), \quad 2\beta_n = \tfrac{1}{2}\nu_n^2\Omega^{-2} \tag{4.5}$$
$$(\alpha_n \simeq \omega_n^2\Omega^{-2}[1 - U_0^2 a^2(8\pi h^3 T\gamma_n^2)^{-1}], \quad 2\beta_n \simeq U_0^2(8\pi\mu h^3\Omega^2)^{-1}, \quad \gamma_n h a^{-1} \ll 1)$$

Equations (3.5) can be reduced to the form of a Mathieu equation [6] (the dots again denote differentiation with respect to the argument $\tau$)

$$\ddot{V}_n + (\alpha_n - 2\beta_n \cos 2\tau)V_n = 0, \quad n = 1, 2, \ldots \tag{4.6}$$

It is natural to assume further that $\beta_n \ll \alpha_n$. As follows from (4.5) this assumption holds for sufficiently large $n$ since $\alpha_n \sim n^2$, $\beta_n \sim n$ as $n \to \infty$. This enables us to use the methods of perturbation theory and to obtain the conditions for which a loss of stability of the zeroth (or any) solution of Eqs (4.6) occurs. In a small neighbourhood of the resonance values of the parameters $\alpha_n$ corresponding to the fundamental or higher resonance zones, the conditions for exponential instability in the plane of the parameters $\alpha_n$ have the form of two-sided inequalities [6]. We will write these conditions for the first four resonance zones of the parametric oscillations of a membrane of an arbitrary mode with number $n$, $n = 1, 2, \ldots$. By [6] we have

$$\begin{gathered}
1 - \beta_n - \frac{1}{8}\beta_n^2 + \frac{1}{64}\beta_n^3 < \alpha_n < 1 + \beta_n - \frac{1}{8}\beta_n^2 + \frac{1}{64}\beta_n^3 \\
4 - \frac{1}{12}\beta_n^2 + \frac{5}{13824}\beta_n^4 < \alpha_n < 4 + \frac{5}{12}\beta_n^2 - \frac{763}{13824}\beta_n^4 \\
9 + \frac{1}{16}\beta_n^2 - \frac{1}{64}\beta_n^3 < \alpha_n < 9 + \frac{1}{16}\beta_n^2 + \frac{1}{64}\beta_n^3 \\
16 + \frac{1}{30}\beta_n^2 - \frac{317}{864000}\beta_n^4 < \alpha_n < 16 + \frac{1}{30}\beta_n^2 + \frac{433}{864000}\beta_n^4
\end{gathered} \tag{4.7}$$

The left-hand and right-hand sides of inequalities (4.7) are power series of $\beta_n$ in which we have retained terms no higher than the fourth power. These inequalities define the resonance zones of excitation of parametric oscillations. Note that their width is of the order of $\beta(k)$, where $k$ is the number of the zone, i.e. it becomes extremely narrow when $k \gg 1$, $\beta_n < 1$.

Specifying the boundaries of the regions of instability in the neighbourhood of different resonance zones in the form of the two-sided inequalities (4.7) may not be convenient for large $n \gg 1$, since $\beta_n \to \infty$ as $n \to \infty$. Hence, instead of $\beta_n$ (4.5) it is preferable to introduce the parameters $\xi_n$ so that Eqs (4.6) take the form

$$\ddot{V}_n + \alpha_n(1 + \xi_n \cos 2\tau)V_n = 0$$
$$\beta_n = -\tfrac{1}{2}\xi_n\alpha_n, \quad \xi_n = -\tfrac{1}{2}\nu_n^2\omega_n^{-2}(1 - \tfrac{1}{2}\nu_n^2\omega_n^{-2}) \to 0, \quad n \to \infty$$

Substituting $\beta_n$ into (4.7) and solving the inequalities for $\alpha_n$ successively in powers of the small quantity $\xi_n$, we obtain the conditions for instability that are equivalent to those indicated and that are more convenient for use when $n \gg 1$, since $\xi_n \sim n^{-1} \to 0$ as $n \to \infty$.

Consider the main resonance zone $k=1$ (the first two-sided inequality of (4.7)). In this range of values of the parameters, having a width $2\beta_n$, an exponential increase in the amplitude of the oscillations of the $n$th mode occurs. The frequency of the oscillations is close to the frequency $\Omega$ of alternation of the electric field; the difference is a quantity of the order of $\beta_n$ (4.5). The growth increment of the amplitude is equal to $1/2\beta_n$ [6].

Resonance zones of higher orders $k \geq 2$ can be investigated in the same way. In these zones an exponential increase in the amplitude of the oscillations with a frequency close to $(k\Omega)$ occurs. The width of the resonance zone (as pointed out above), the difference in the frequency and the growth increment of the amplitude are quantities of the order of $\beta_n^k$. In practice [8], parametric excitation of oscillations in the fundamental resonance zone $k=1$ (4.7) occurs most easily for the lower modes of oscillation $n=1, 2, \ldots, n^*$ where $n^*$ "is not very large".

If the natural frequency of a fixed mode $n^*$ satisfies one of the conditions (4.7), but none of the remaining modes $n \neq n^*$ satisfy any of the above two-sided inequalities, only this mode $n^*$ will be excited. Otherwise several modes of oscillations may be excited in different resonance zones.

Satisfaction of the conditions of instability (4.7) of the parametric oscillations (4.6) leads to an exponential unlimited increase in the amplitude of the oscillations as $t \to \pm\infty$ [6]. This can be explained by the incomplete linear model of the oscillations of the membrane. If there is a non-linearity and dissipation in the system, the stationary oscillations of limited amplitude may set in. Sources of non-linearity can be the following.

1. Higher powers of $\varepsilon$ (i.e. of the variable $V$) are taken into account in the expressions for the potentials $u_{1,2}$ see Section 2), i.e. the geometrical non-linearity is taken into account when calculating the ponderomotive forces of the electric field.

2. The presence of a non-linear relationship between the tension $T$ of the membrane and the displacement (physical non-linearity of the material).

3. The geometrical non-linearity is taken into account when calculating the elastic forces which return the elements of the membrane to the position of equilibrium (the extensibility of the material is taken into account).

The effect of one or several types of the above (and possibly other) non-linearities leads to a limitation on the amplitude of parametrically excited oscillations, and also leads to the possibility of the existence of periodic stationary oscillations. Note that the presence of considerable dissipation may lead to stable stationary oscillations in a resonance zone both in the non-linear and linear approaches.

### 4.3. *Conclusions*

Using the results obtained in Section 4.1 we have established the possibility of a loss of stability of the position of equilibrium of a membrane in a constant electric field. It is technically possible to carry out laboratory experiments to determine the shift in the natural frequencies of the oscillations of the membrane in a constant electric field, and also to observe the loss of stability of the equilibrium state $V \equiv 0$.

Parametric excitation of the oscillations of a membrane placed between grounded plates of a capacitor and subjected to the action of an alternating electric field is possible in principle. The excitation of the first mode in the fundamental resonance zone by an appropriate choice of the parameters of the system, such as $a, h, U_0, \Omega, T, \mu$ (see Section 4.2) is technically the simplest.

Note that the problem of the parametric oscillations of an elastic membrane in an alternating electric field considered above is in a certain sense the dual problem of the mechanical excitation of an electric current considered in [9], where it was shown that it is possible to excite electric oscillations in a circuit by a periodic variation of the capacitance.

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